# PENSIONS

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## 1. Defined Benefit Pensions

In this note we compute the requirements for Defined Benefit pensions, where the pension is a function of final salary and length of service. The numbers are chosen to mimic USS before 2016, when it was an undiminished final salary scheme: it is salutary to remember the pension we enjoyed so recently. The idea is solely to provide a simple (even simplistic) model for experiment, the focus being the finance: all actuarial details are therefore ignored (except for retirement years before death!).

We assume that a scheme member begins with an initial income of 1 at time zero, which grows at rate $g$ until retirement at time $R$. The member saves at contribution rate $c$ at interest rate $r$, so the pension fund at retirement is given by

$$F = c \int_0^R e^{gt} e^{r(R-t)}\, dt \tag{1}$$

if we assume continuous time, for simplicity. Thus

$$F = c \left( \frac{e^{gR} - e^{rR}}{g-r} \right) = c e^{gR} \left( \frac{1 - e^{-(g-r)R}}{g-r} \right). \tag{2}$$

At retirement, the aim is to pay the member a lump sum of $1.5 \exp(gR)$, followed by a pension of $(R/80)\exp(gR)$ until death $D$ years later. This is where the true actuarial difficulties begin, but we shall avoid this by treating $D$ as a parameter. We use the fund to create an annuity to pay the pension, and a standard argument implies the pension cost

$$P = \frac{3}{2} e^{gR} + \frac{R}{80} e^{gR} \int_0^D e^{-rt}\, dt = e^{gR} \left( \frac{3}{2} + \frac{R}{80} \left[ \frac{1 - e^{-rD}}{r} \right] \right). \tag{3}$$

Equating (2) and (3), and dividing by $\exp(gR)$, we obtain

$$c \left( \frac{1 - e^{-(g-r)R}}{g-r} \right) = \frac{3}{2} + \frac{R}{80} \left[ \frac{1 - e^{-rD}}{r} \right] \tag{4}$$

or

$$c = \frac{\frac{3}{2} + \frac{R}{80} \left( \frac{1-e^{-rD}}{r} \right)}{\left( \frac{1-e^{-(g-r)R}}{g-r} \right)}. \tag{5}$$

How do we choose the parameters? I have chosen $r = 0.02$ in most experiments, although USS sometimes boasts of much higher yields in the equity investments. The reason for the low value is that much of the USS fund is in UK bonds, whose yield has been near zero since the financial crisis of 2007–2008: it is the near-zero yield, imposed by central banks as part of Quantitative Easing, which is at the root of the problem for all pension funds. You will also see further reasons for this choice in the examples below.

I have provided MATLAB code at the end of this Section for experimentation. As we should all know, the fund imposed a £55K cap on its Defined Benefit scheme in 2016, and this was probably necessary (see the examples), and it's easy to alter the code to include caps. Some of my experiments strongly suggest that £55K is not sustainable while $r$ remains so low, unless contributions increase substantially. The alternative is a lower cap, e.g. £45K, which does seem feasible. Of course, the present decision is to remove the Defined Benefit component entirely in April 2019. The usefulness of caps is further suggested by the next simple result.

**Lemma 1.1.** *The contribution rate c is an increasing function of the salary increase rate $g$.*

*Proof.* If we let $v(g)$ denote the denominator of (5) as a function of the salary increase rate $g$, then we can write this as the integral

$$v(g) := \frac{1 - e^{-(g-r)R}}{g - r} = \int_0^R e^{-(g-r)s}\, ds.$$

We can now differentiate under the integral sign with respect to $g$, obtaining

$$\frac{\partial v}{\partial g} = -\int_0^R s e^{-(g-r)s}\, ds < 0.$$

Thus $v(g)$ is a strictly decreasing function of $g$, which implies that $c(g)$, being a multiple of its reciprocal, is a strictly increasing function of $g$. □

Here are some examples of the MATLAB code's output for different parameters.

**Example 1.1.** *For an academic who triples their salary over* 40 *years, here is one result.*

```
Interest rate r = 0.02, service length R = 40 years
Final salary/Initial salary = exp(g*R) = 3
The contribution percentage needed for D = 20 funded years c = 0.281721
The number of funded years for the USS contribution rate is 17.8077
```

Example 1.1 illustrates several USS problems, not least of which is the high required value of $c$. At present, each month USS members contribute 8% of gross salary, whilst the employer contributes 18%. The choices are to increase $r$, fund higher contributions, reduce $g$ (effectively capping the salary) or reduce $D$ (effectively ending Defined Benefit, unless our VCs introduce a euthanasia component).

**Example 1.2.** *We can model a salary cap by imposing* $\exp(gR) = 2$. *If we assume an initial salary of £27K, then this is close to our current salary cap of £55K.*

```
Interest rate r = 0.02, service length R = 40 years
Final salary/Initial salary = exp(g*R) = 2
The contribution percentage needed for D = 20 funded years c = 0.23077
The number of funded years for the USS contribution rate is 23.8243
```

*This is better news than Example 1.1.*

**Example 1.3.** *We can model a more stringent salary cap by imposing* $\exp(gR) = 1.7$. *If we assume an initial salary of £27K, then this is close to a salary cap of £45K.*

```
Interest rate r = 0.02, service length R = 40 years
Final salary/Initial salary = exp(g*R) = 1.7
The contribution percentage needed for D = 20 funded years c = 0.212218
The number of funded years for the USS contribution rate is 27.0143
```

Finally, some of our colleagues see *much* larger salary increases during their career. The following example is designed for these successful individuals.

**Example 1.4.** *Let us consider a highly successful academic who increases their salary from £27K to £400K:*

```
Interest rate r = 0.02, service length R = 40 years
Final salary/Initial salary = exp(g*R) = 14.8148
The contribution percentage needed for D = 20 funded years c = 0.543296
The number of funded years for the USS contribution rate is 6.76152
```

Until the mid-1990s, it was still usual for most academics to remain lecturers throughout their academic careers, with a small minority achieving higher rank. Thus the typical value of $g$ was smaller, say $\exp(gR) = 2$ for $R = 40$, or $g \approx 0.017$. In other words, limited salary progression is good for Defined Benefit schemes based on final salary. For that reason, the SAUL pension fund (Superannuation Arrangements of the University of London) is much more likely to be able to keep funding a Defined Benefit scheme, since it is mostly limited to lower salary ranges.

**Example 1.5.** *Why have I chosen the relatively small value of* $r = 0.02$ *in my examples. Surely USS is better than that, given the tremendous bonuses paid to their fund managers? Let's try an example:*

```
Interest rate r = 0.04, service length R = 40 years
Final salary/Initial salary = exp(g*R) = 2
The contribution percentage needed for D = 20 funded years c = 0.128724
The number of funded years for the USS contribution rate is 36.2423
```

*This would be a world in which USS pension benefits should increase not decrease! Clearly this is not our world, so USS is achieving something in the range* $0.01 \le r \le 0.03$.

**Example 1.6.** *As a simple check on our calculations, let us consider the special case when* $g$ *and* $r$ *are tiny. In this case, Taylor expansions of the exponential yield*

$$cR = \frac{3}{2} + \frac{RD}{80},$$

*or*

$$c = \frac{3}{2R} + \frac{D}{80}.$$

*If we consider an academic who retires after* $R = 40$ *years of service, then*

$$c = \frac{3 + D}{80}.$$

*Thus* $D = 20$ *years of retirement require* $c = 23/80 \approx 29\%$ *of gross salary, as expected.*

Finally, on the next page there is the promised MATLAB to calculate $c$ and $D$.

```
%
% Defined Benefit Pensions
%
% r is the interest rate
%
r=0.02;
%
% R = number of years of service, exp(g*R)=3.0
%
R=40;
%
% Assumption: Final salary/initial salary =  exp(gR)
%
% g=log(3.0)/R;
g=log(2.0)/R;
% g = log(1.7)/R;
%g = log(400/27)/R;
%
% D = number of years in retirement (i.e. before death)
% Employer pays 18% and employee pays 8% of gross salary
%
%
D=20;
c=0.26;
%
printf('Interest rate r = %d, service length R = %d years\n', r, R)
printf('Final salary/Initial salary = exp(g*R) = %d\n', exp(g*R))
%
% F1 is multiplied by c to obtain the actual
% pension fund total
%
F1=(1-exp(-(g-r)*R))/(g-r);
%
% We first find the annuity base cost required
% for D years of retirement, together with the
% corresponding contribution rate.
%
P=1.5 + (R/80)*(1-exp(-r*D))/r;
c1=P/F1;
%
printf('The contribution percentage needed for D = %d funded years c = %d\n', D, c1)
%
% We next compute the number D1 of retirement years
% our fund can actually provide for the USS contribution rate c.
%
F=c*F1;
D1=-(1.0/r)*log(1-r*(F-3/2)/(R/80));
printf("The number of funded years for the USS contribution rate is %d\n", D1)
```