Explorations in Mathematics

Due: Monday 8th April 2024

This assignment counts for 50% of the total marks for the module. This assessment covers the material in the units *Sound and Music*, and *Large and Small*. You will have a much better chance of doing well if you re-read the notes carefully, and in particular try the exercises and read through the worked solutions provided, as the questions I'm asking are in a similar spirit to this material.

How to Submit this Assignment You will submit the work online, on the Moodle page for this module. You are welcome to submit handwritten work (I know how long it takes to type mathematics); this is what we expect most people will do. Handwritten work must be scanned and submitted as a pdf. You are responsible for making sure the scan quality is good enough for me to read. You can type all or part of your work if you prefer – however you do it, you'll ultimately need to submit a single pdf for the whole assignment. Detailed guidance about submission is in the Assessment section of the Moodle page. The guidance includes information about recommended apps to scan handwritten documents.

How will marks be awarded? Please look at the marking guidance in the Assessment section of the Moodle page, but the key things are to set out your work clearly, to write legibly, and to **show working** that explains your reasoning. You don't need to write a novel – you can assume I know what the notes say, so there's no need to include large chunks of exposition – but something like "by Identity 2 in the notes" helps me to understand what your reasoning is, and may allow you to get partial credit for a wrong answer. If you just write down the correct answer with no explanation then you won't get full marks. Some of the marks in your coursework will always be for the clarity of your explanations!

Late Submission College rules mean that work submitted late, unless you have a mitigating circumstances claim upheld, will be given a mark penalty. Your mark will be reduced by 10% if it is up to 7 days late. Work more than 7 days late, up to a maximum of 14 days late, will be capped at 40%. Work more than 14 days late will be given a mark of zero. For more information on the mitigating circumstances policy, please visit the Assessment section of the Moodle page for this module.

Music and Sound

MS2: This question is about guitar strings. In order to use Mersenne's Laws (that $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$), we need to know their linear density μ. Suppose a guitar string has volume V metres cubed, mass M kg, (volumetric) density D kg/m^3, length L metres and cross-sectional radius r metres. The volumetric density of the material used to make guitar strings is 7860kg/m^3, so we have $D = 7860$. The volume V of a guitar string is its cross-sectional area multiplied by its length, so $V = \pi r^2 L$. Since density $= \frac{\text{mass}}{\text{volume}}$, we have that mass = density $\times$ volume, and so $M = DV$. Linear density is mass per unit length. Therefore,

$$\mu = \frac{M}{L} = \frac{DV}{L} = D\pi r^2 = 7860\pi r^2.$$

Guitar strings have different thicknesses. The best selling brand of guitar strings is Ernie Ball. In their regular pack, the six strings have gauges 10, 13, 17, 26, 36, 46. The gauge is the thickness, or diameter, of a string. It is measured in thousandths of an inch. That is, a 10-gauge string would have diameter 0.010 inches, and radius 0.005 inches. Now, 1 inch is 0.0254 metres. Hence, the 10-gauge string has radius $0.005 \times 0.0254 = 0.000127$ metres. This question is about tuning a six-string guitar where each string has length $L = 0.63$ metres.

(a) The 10-gauge string produces the highest note on the guitar. Suppose it is adjusted to a tension of 68.8N. Use Mersenne's Laws to find the frequency in Hertz of the note it plays, to 1 decimal place. You will first need to find the linear density using the formula $\mu = 7860\pi r^2$ above. [3]

(b) The standard tuning for the highest string on the guitar is an E with frequency 329.6Hz. What is the difference in cents between this frequency and the frequency you obtained in part (a)? [2]

(c) The 46-gauge string produces the lowest note. It should be tuned to an E two octaves below the high E, with frequency 82.4Hz. Find the linear density, and hence use Mersenne's Laws to determine the correct tension in this string, to produce the required note. [5]

Large and Small

LS2: A grey squirrel is roughly 8 times smaller than a human, on average (excluding the tail), but is otherwise not too dissimilar. Compare their height, weight, strength, food consumption, jumping ability compared to ours, based solely on the change in length scale. Clarity of assumption and argument is required, although there are many possible ways in which to obtain full marks. [10]