11.

PREDICTOR: Let $h=1$ and consider the linear functional

$$L_p(y) = y(3) + \frac{3}{2}y(2) - 3y(1) + \frac{1}{2}y(0) - 3y'(2),$$

y	$L_p(y)$
1	$1 + \frac{3}{2} - 3 + \frac{1}{2} = 0$
x	$3 + 3 - 3 - 3 = 0$
x^2	$9 + 6 - 3 - 12 = 0$
x^3	$27 + \frac{3}{2}\cdot 8 - 3 - 3.3.2^2 = 27+12 - 3 - 36 = 0$

Hence $\operatorname{Ker} L_p \supseteq \mathbb{P}_3$ and y_p is 3^{rd} order.

[Or just Taylor expand.]

CORRECTOR: Set $h=1$ & define

$$L_c(y) = y(3) - \frac{18}{11}y(2) + \frac{9}{11}y(1) - \frac{2}{11}y(0) - \frac{6}{11}y'(3)$$

y	$L_c(y)$
1	$1 - \frac{18}{11} + \frac{9}{11} - \frac{2}{11} = 0$
x	$3 - \frac{18}{11}.2 + \frac{9}{11} - \frac{6}{11} = \frac{1}{11}(33 - 36 + 3) = 0$
x^2	$9 - \frac{18.4}{11} + \frac{9}{11} - \frac{6}{11}.6$

$$= \frac{1}{11}\left(99 - 72 + 9 - 36\right)$$

$$= \frac{1}{11}\left(108 - 108\right) = 0.$$

x^3

$$27 - \frac{18.8}{11} + \frac{9}{11} - \frac{6}{11}.27$$

$$= \frac{5.27 - 144 + 9}{11} = \frac{135 + 9 - 144}{11} = 0.$$

Hence $L_c(y) = 0$ for $y \in \mathbb{P}_3$ & y_c is 3^{rd} order.

ERROR ESTIMATE: Direct expansion or setting $y = x^4$ (or $y = (x-1)^4$) (or $y = (2x-3)^4$) in L_p and L_c give local truncation errors

$$y(t_{n+3}) - y^p_{n+3} = \frac{1}{4} h^4 y^{(4)}(t_n) + O(h^5) \qquad (1)$$

and

$$y(t_{n+3}) - y^c_{n+3} = -\frac{3}{22} h^4 y^{(4)}(t_n) + O(h^5). \qquad (2)$$

Then $(2) - (1)$ gives

$$y^p_{n+3} - y^c_{n+3} = \left(-\frac{3}{22} - \frac{1}{4}\right) h^4 y^{(4)}(t_n) + O(h^5) \qquad (3)$$

and substituting in (2) gives

$$y(t_{n+3}) - y^c_{n+3} = \underbrace{\left(\frac{-\frac{3}{22}}{-\frac{3}{22} - \frac{1}{4}}\right)}_{\frac{3/22}{\frac{3}{22} + \frac{1}{4}} = \frac{3}{3 + \frac{11}{2}} = \frac{3}{\frac{6+11}{2}} = \frac{6}{17}.} \left(y^p_{n+3} - y^c_{n+3}\right) + O(h^5)$$