

$$9. \quad y_{n+2} - \frac{4}{3} y_{n+1} + \frac{1}{3} y_n = \frac{2}{3} h f(t_{n+2}, y_{n+2})$$

so $y' = \lambda y$ implies

$$y_{n+2} - \frac{4}{3} y_{n+1} + \frac{1}{3} y_n = \frac{2}{3} h \lambda y_{n+2}$$

or

$$\left(1 - \frac{2}{3} h \lambda\right) y_{n+2} - \frac{4}{3} y_{n+1} + \frac{1}{3} y_n = 0$$

$$\text{i.e. } (3 - 2z) y_{n+2} - 4y_{n+1} + y_n = 0 \quad \text{where } z = h\lambda.$$

Now $z \in \partial D \Rightarrow y_n = e^{in\theta}$ for some $\theta \in \mathbb{R}$, i.e.

$$(3 - 2z) e^{i(n+2)\theta} - \frac{4}{3} e^{i(n+1)\theta} + \frac{1}{3} e^{in\theta} = 0$$

$$\text{or } (3 - 2z) e^{2i\theta} - 4e^{i\theta} + 1 = 0$$

$$\text{Hence } 2z e^{2i\theta} = 3e^{2i\theta} - 4e^{i\theta} + 1$$

or

$$z = \frac{1}{2} (3 - 4e^{-i\theta} + e^{-2i\theta})$$

We want $D \cap \mathbb{R}$ so consider

$$\operatorname{Re} z = \frac{1}{2} (3 - 4\cos\theta + \cos 2\theta)$$

$$= \frac{1}{2} (3 - 4\cos\theta + 2\cos^2\theta - 1)$$

$$= 1 - 2\cos\theta + \cos^2\theta$$

$$= (\cos\theta - 1)^2$$

$$\geq 0.$$

Hence $D \cap \mathbb{R} \supseteq (-\infty, 0)$ and A 's A -stable.