

Mathematical Tripos Part IB: Lent Term 2024

Numerical Analysis

1. Course description

1. Polynomial approximation

Polynomial interpolation. Lagrange and Newton polynomials. Divided differences.

2. Error bounds for polynomial interpolation. Chebyshev polynomials. Optimal interpolation knots.
3. Orthogonal polynomials. Three-term recurrence relation. Least squares polynomial fitting.
4. Approximation of linear functionals. Numerical integration. Numerical differentiation.
5. Error of approximation of linear functionals. The Peano kernel theorem. Sharp error bounds.

6. Ordinary differential equations

Basics. Euler method. Backward Euler. Trapezoidal rule.

7. Multistep methods. Order, convergence. Root condition. Dalquist equivalence theorem.
8. Construction of multistep methods. Adams methods. Backward differentiation formulas (BDF). Relation to numerical quadrature and numerical differentiation formulas.
9. Runge-Kutta methods. Explicit and implicit RK methods. Order of the RK methods.
10. Stiff ODEs. Linear and A-stability. Stability of multistep methods. Stability of Runge-Kutta methods.
11. Numerical implementation. The Milne device. Predictor-corrector scheme. Embedded Runge-Kutta methods.

12. Numerical linear algebra.

LU factorization. Pivoting.

13. Existence and uniqueness of the LU factorization. Symmetric positive definite matrices. Diagonally dominant matrices. Sparse and band matrices.
14. QR factorization. Orthogonal matrices. The Gram-Schmidt orthogonalization.
15. Givens rotations. Householder reflections.
16. Linear least squares. Normal equations. QR and least squares.

2. Lecture notes

- ★ At the beginning of each lecture new handouts will be distributed. Spare handouts of the previous lectures will be available from myself (after the lectures).

In reasonable circumstances (illness, no spare copies left, etc.), I will send handouts via email.

The lecture notes may appear on the web after the term.

3. Appropriate books

1. (Polynomial approximation)
S. D. Conte, C. de Boor, Elementary numerical analysis: an algorithmic approach, McGraw-Hill 1980
2. (ODEs)
A. Iserles, A first course in the numerical analysis of differential equations, Cambridge University Press, 2009
3. (Numerical linear algebra)
No single book known to me.

- ★ For all three parts, the lectures and the lecture notes are the best source.

4. Communication

- **E-mail address:** <a.shadrin@damtp.cam.ac.uk>

5. What is Numerical Analysis

- Numerical analysis is studying practical algorithms based on solid theoretical background, so in a sense builds a bridge between pure and applied mathematics.

Its main goal is to construct a finite mathematical model that approximates some infinite process with a given accuracy. It is also dealing with the questions of effectiveness, complexity and stability of algorithms that lie in the heart of numerical simulation of reality. Examples include:

- 1) Approximation: reconstruct f from n bits of information;
- 2) ODEs, PDEs: $\mathbf{y}' = f(t, \mathbf{y})$;
- 3) Algebraic equations: solve $f(\mathbf{x}) = 0$, $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$, say, linear systems $A\mathbf{x} = \mathbf{b}$;
- 4) Optimization: find $\min_{\mathbf{x} \in \Omega} f(\mathbf{x})$;
- 5) etc.