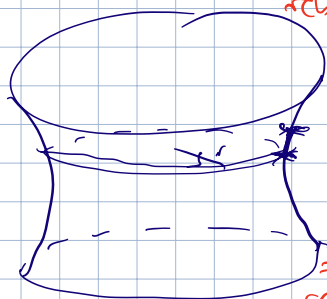


If $r = r(z)$, then area

$$A = 2\pi \int_a^b \underbrace{r \sqrt{1+(r')^2}}_{f(z, r, r')} dz.$$



Then $\frac{\partial f}{\partial z} = 0 \Rightarrow f - r' \frac{\partial f}{\partial r'} = \text{const.}$

$$r \sqrt{1+(r')^2} - (r')^2 \cdot r (1+(r')^2)^{-\frac{1}{2}} = c.$$

So

$$\frac{c}{r} = \sqrt{1+(r')^2} - \frac{(r')^2}{\sqrt{1+(r')^2}}$$

$$= \frac{1+(r')^2 - (r')^2}{\sqrt{1+(r')^2}}$$

i.e.

$$\frac{c}{r} = \frac{1}{\sqrt{1+(r')^2}}$$

Hence $1+(r')^2 = \frac{r^2}{c^2}$

$$r' = \pm \sqrt{\frac{r^2}{c^2} - 1}$$

$$\int_{r(0)=c}^{r(z)} \frac{dr}{\sqrt{\frac{r^2}{c^2} - 1}} = z$$

Now $r = c \cosh u$

$$\Rightarrow \text{LHS} = \int_{u=0}^{\cosh^{-1} \frac{r(z)}{c}} c \, du = z$$

$(r(0)=c \Rightarrow \cosh u=1 \Rightarrow u=0)$ where

CLA m:

$$\frac{r}{c} = \cosh \frac{z}{c} \quad \frac{a}{c} = \cosh \frac{b}{c}.$$

$$\therefore \cosh^{-1} \frac{r(z)}{c} = \frac{z}{c}$$

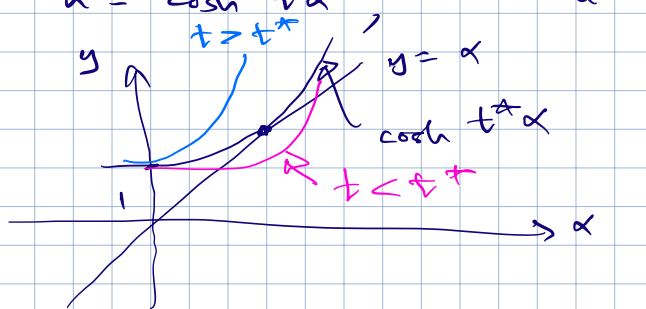
$$\text{or} \quad \frac{r(z)}{c} = \cosh \frac{z}{c}$$

$$\text{Finally, } z=b \rightarrow \frac{r(b)}{c} = \cosh \frac{b}{c}$$

$$\therefore \frac{a}{c} = \cosh \frac{b}{c}$$

$$\text{If } b = t a, \text{ then } \frac{a}{c} = \cosh t \frac{a}{c}$$

$$\therefore \alpha = \cosh t \alpha \quad \alpha = \frac{a}{c}$$



Here $\cosh t \alpha - \alpha > 0$ for $\alpha > 0$
for $t > t^*$ (see picture)